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On the Numerical Solution of Thermal Shock Problem for Generalized Magneto-Thermoelasticity for an Infinitely Long Annular Cylinder with Variable Thermal Conductivity

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In the present work, we study the transient phenomena in magneto-thermoelastic model in the context of the theory of generalized thermoelasticity GL model with variable thermal conductivity. The numerical solutions for the displacement, temperature, and radial and hoop streasses in the context of FEM are obtained. The boundary conditions for the mechanical and Maxwell's stresses at the internal and outer surfaces are considered. An application of an infinitely long annular cylinder is investigated for the inner surface is traction free and subjected to thermal shock, while the outer surface is traction free and thermally isolated. Finally, the displacement, incremental temperature, the stress components are obtained and then presented graphically to show the influence of the variables on the phenomena.

Keywords: Numerical Solution, Thermal Shock, Relaxation Times, Annular Cylinder, Magnetic Field, GL Model.

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1. INTRODUCTION

In recent years, more attentions have been made for the theory thermal shock problem of generalized thermoelasticity because of its utilitarian aspects in diverse fields, especially, Engineering, Structures, Geology, Biology, Geophysics, Acoustics, Physics, Plasma, etc. Duhamel[1] and Neumann[2] introduced the theory of uncoupled thermoelasticity. The theory of elasticity with nonuniform heat which was in half-space subjected of thermal shock in this context which known as the theory of uncoupled thermoelasticity and the temperature is governed by a parabolic partial differential equation in temperature term only has been discussed.[3] Biot[4] introduced the theory of classical thermoelasticity, the equation of motion is hyperbolic in nature, whereas the heat conduction equation is parabolic in nature; the theory predicts a finite speed for predominantly elastic disturbances but an infinite speed for predominantly thermal disturbances, which are coupled together. Obviously, this result is physically unrealistic, so,[5–11] made an experimental investigations conducted on various solids, for example, have shown that heat pulses do propagate with finite speed. These theories remove the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity introduced by Biot.[4] Lord and Shulman,[12] have discovered the theory which determines the finite speed for the motion due to thermal field using one relaxation time. By including temperature rate, Green and Lindsay[13] violated the classical Fourier's law of heat conduction when the body under consideration has a center of symmetry. This theory also predicts a finite speed of heat propagation using two relaxation times. This implies that the thermal wave propagates with infinite speed, a physically impossible result.

During the second half of twentieth century, nonisothermal problems of the theory of elasticity became increasingly impact. This is due mainly to their many applications in widely diverse fields. First, in the nuclear field, the external high temperatures and temperature gradients

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originating inside nuclear reactors influence their design and operations. Secondly, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure.[14] Nowacki[15] investigated the dynamic problems of thermoelasticity. Some problems of thermoelasticity are discussed.[16,17] Dhaliwal and Sherief[18] discussed three different models of thermoelasticity in an alternative way including the anisotropic case.[18] A survey article of representative theories in the range of generalized thermoelasticity is due to Hetnarski and Ignaczak.[19] Eraby and Suhubi[20] studied wave propagation in a cylinder. Ignaczak[21] studied a strong discontinuity wave and obtained a decomposition theorem.[22] Ezzat[23] has also obtained the fundamental solution for this theory. Many problems have been solved in the context of the generalized thermoelasticity by El-Maghraby and Yousef,[24,25] Youssef et al.[26] and Yousef.[27–29] Noda[30] investigated the thermal stresses in materials with temperature-dependent properties. Modern structural elements are often subjected to temperature changes of such magnitude that their material properties may no longer be regarded as having constant values even in an approximate sense. The thermal and mechanical properties of materials vary with temperature, so that the temperature dependence of material properties must be taken into consideration in the thermal stress analysis of these elements.[31–33]

In recent years, the theory of magneto-thermoelasticity which deals the interactions among strain, temperature and electromagnetic fields has drawn the attention of many researchers because of its extensive uses in diverse fields, such as Geophysics for understanding the effect of the Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, development of a highly sensitive superconducting magnetometer, electrical power engineering, optics, etc. Knopoff[34] and Chadwick[35] studied these types of problems in the beginning and developed by Kaliski and Petykiewicz.[36] The generalized magneto-thermoelasticity in a perfectly conducting medium is investigated.[37] Baksi et al.[38] illustrate magneto-thermoelastic problems with thermal relaxation and heat sources in a three dimensional infinite rotating elastic medium. In Yousef and Abbas,[39] the influence of variable thermal conductivity, thermal shock and relaxation time for an annular cylinder has been discussed. Tianhu et al.[40] studied the two-dimensional generalized thermal shock problem for a half-space in electromagneto-thermoelasticity. Abd-Alla et al.[41] and Abo-Dahab and Mohamed[42] illustrated some problems in magneto-thermoelasticity and viscosity. Abo-Dahab and Abbas[43] illustrated LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity. Recently, Refs. [44–47] studied other problems in waves.

The present paper is devoted to estimate the influence of relaxation time, magnetic field, thermal shock and variable thermal conductivity, GL model of generalized thermoelasticity is considerd under variable thermal conductivity and magnetic field. We consider an infinitely long annular cylinder whose inner surface is traction free and subjected to thermal shock and magnetic field. The outer surface is also traction free and thermally isolated. The medium parameters quiescent initial state. The FEM is proposed to obtain the displacement, temperature and the radial and hoop stresses. Finally, the results obtained are represented graphically.

2. GOVERNING EQUATIONS

The constitutive equations

$$\sigma_{ij} = \left[\lambda e_{kk} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta\right]\delta_{ij} + 2\mu e_{ij} \tag{1}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

The Maxwell's stress equation

$$\tau_{ij} = \mu_e[H_i h_j + H_j h_i - H_k \cdot h_k \delta_{ij}] \tag{3}$$

The equation of motion is

$$\sigma_{ji,j} + F_i = \rho \ddot{u}_i \tag{4}$$

where

$$F_i = (\vec{J} \times \vec{B})_i$$

which tends to

$$\mu u_{i,jj} + (\lambda+\mu)\, u_{j,ij} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta_{,i} + F_i = \rho\ddot{u}_i \tag{5}$$

wich reduces to

$$\begin{aligned}\mu(u_{r,rr} + u_{r,zz} + u_{r,\theta\theta}) + (\lambda+\mu)(u_{r,rr} + u_{z,rz} + u_{\theta,r\theta})\\ -\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta_{,r} + F_r = \rho\ddot{u}_r\end{aligned} \tag{6}$$

In r coordinate, Eq. (6) becomes

$$\mu u_{r,rr} + (\lambda+\mu) u_{r,rr} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta_{,r} + F_r = \rho\ddot{u}_r \tag{7}$$

for slowly moving medium, the variation of magnetic field and electric field are given by Maxwell's equations as the form

$$\begin{gathered}\operatorname{curl}\vec{h} = \vec{J}, \quad \operatorname{curl}\vec{E} = -\mu_e\dot{\vec{h}}, \quad \vec{E} = -\mu_e(\dot{\vec{u}} \times \vec{H})\\ \operatorname{div}\vec{h} = 0, \quad \operatorname{div}\vec{E} = 0,\\ \vec{H} = \vec{H}_o + \vec{h}(r,t), \quad \vec{H}_o = (0, H_o, 0)\end{gathered} \tag{8}$$

if the medium is unbounded, the current density $\vec{J}$ is obtained by the Ohm's law as

$$\vec{J} = \sigma\left(\vec{E} + \frac{\partial \vec{u}}{\partial t} \times \vec{B}\right) \tag{9}$$

The equation of heat conduction takes the form

$$K\theta_{,kk} = \rho C_e\left(1+\tau_o\frac{\partial}{\partial t}\right)\dot{\theta} + \gamma T_o\left(1+\tau_o\delta\frac{\partial}{\partial t}\right)\dot{e} \tag{10}$$

which can be written in the form

$$K\theta_{,kk} = \frac{K}{k}\left(1+\tau_o\frac{\partial}{\partial t}\right)\dot{\theta} + \gamma T_o\left(1+\tau_o\delta\frac{\partial}{\partial t}\right)\dot{e} \tag{11}$$

where

$$\rho C_e = \frac{K}{k} \tag{12}$$

We will use the mapping:[48]

$$\vartheta = \frac{1}{K_o}\int_0^{\theta} K(\theta')\,d\theta' \tag{13}$$

By differentiating Eq. (13) with respect to x, we obtain

$$K_o\vartheta,_i = K(\theta)\theta,_i \tag{14}$$

By differentiating a gain Eq. (14) with respect to x, we get

$$K_o\vartheta,_{ii} = [K(\theta)\theta,_i],_i \tag{15}$$

With the same manner, by differentiating the mapping with respect to time, we have

$$K_o\dot{\vartheta} = K(\theta)\dot{\theta} \tag{16}$$

Hence, the heat equation will take the form

$$\vartheta,_{ii} = \left(1+\tau_o\frac{\partial}{\partial t}\right)\frac{\dot{\theta}}{k} + \frac{\gamma T_o}{K_o}\left(1+\tau_o\delta\frac{\partial}{\partial t}\right)\dot{e} \tag{17}$$

Now we will take the thermal conductivity as a function of the temperature with linear form as follows:[48]

$$K = K(\theta) = K_o(1+K_1\theta) \tag{18}$$

Then, we have from the last equation and the mapping the following forms

$$\vartheta = \theta + \frac{K_1}{2}\theta^2 \tag{19}$$

$$\vartheta_{,i} = \theta_{,i}(1+K_1\theta) \tag{20}$$

and

$$\theta = \frac{-1+\sqrt{1+2K_1\vartheta}}{K_1} \tag{21}$$

Substituting from Eq. (21) into Eq. (5), we get

$$(\lambda+\mu)u_{j,ij} + \mu u_{i,jj} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right) \times \frac{\vartheta_{,i}}{\sqrt{1+2K_1\theta}} + F_i = \rho\ddot{u}_i \tag{22}$$

The constituitive relation takes the form

$$\sigma_{ij} = \left(\lambda e_{kk} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right) \times \left[\frac{-1+\sqrt{1+2K_1\vartheta}}{K_1}\right]\right)\delta_{ij} + 2\mu e_{ij} \tag{23}$$

3. FORMULATION OF THE PROBLEM

We consider an infinitely long annular cylinder whose inner surface is traction free and subjected to a thermal shock, while the outer surface also is traction free but thermally isolated. We assume also that there is heat sources acting in the medium. We use a cylindrical system of coordinates (r, ψ, z) with the z-axis lying along the axis of the cylinder. Due to symmetry, the problem is one-dimensional with all the functions considered depending on the radial distance r and the time t where $R_1 \le r \le R_2$.

The displacement vector has the components

$$u_r = u(r,t), \quad u_\psi(r,t) = u_z(r,t) = 0 \tag{24}$$

The heat conduction Eq. (17) takes the form

$$\nabla^2\vartheta = \left(1+\tau_o\frac{\partial}{\partial t}\right)\frac{\dot{\vartheta}}{k} + \frac{\gamma T_o}{K_o}\left(1+\tau_o\delta\frac{\partial}{\partial t}\right)\dot{e} \tag{25}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

The equation of motion has the form

$$(\lambda+2\mu+\mu_e H_o^2)\frac{\partial e}{\partial r} - \frac{\gamma}{\sqrt{1+2K_1\vartheta}}\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\vartheta}{\partial r} = \rho\ddot{u} \tag{26}$$

where

$$e = \frac{1}{r}\frac{\partial(ru)}{\partial r}$$

the constitutive equations take the forms

$$\sigma_{rr} = \lambda e + 2\mu\frac{\partial u}{\partial r} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right) \times \left(\frac{-1+\sqrt{1+2K_1\vartheta}}{K_1}\right) \tag{27}$$

$$\sigma_{\psi\psi} = \lambda e + 2\mu\frac{u}{r} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right) \times \left(\frac{-1+\sqrt{1+2K_1\vartheta}}{K_1}\right) \tag{28}$$

$$\sigma_{zz} = \lambda e - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\left(\frac{-1+\sqrt{1+2K_1\vartheta}}{K_1}\right) \tag{29}$$

$$\sigma_{rz} = \sigma_{\psi r} = \sigma_{z\psi} = 0 \tag{30}$$

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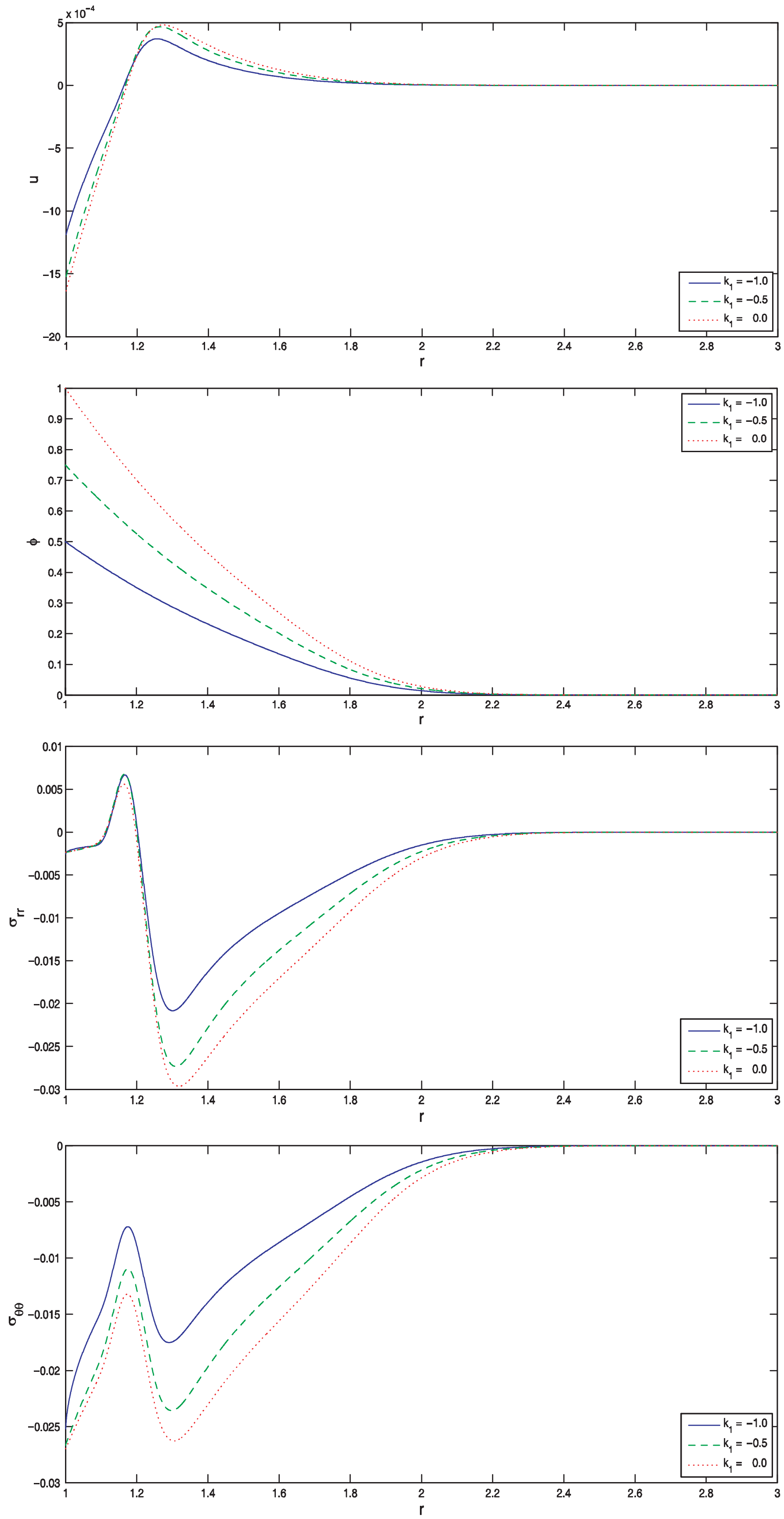

Fig. 1. Continued.

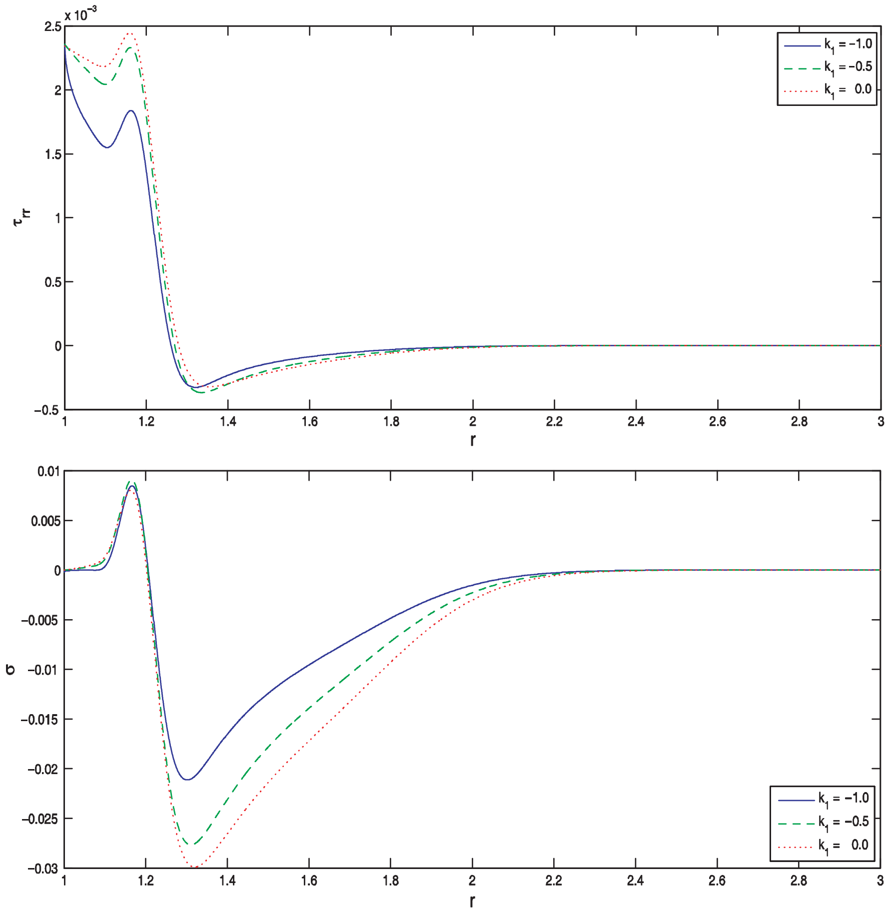

Fig. 1. Effect of the variable conduction on u, $\phi = \vartheta$, σ_{rr}, $\sigma_{\theta\theta}$, τ_{rr} and $\sigma = \sigma_{rr} + \tau_{rr}$ when $\tau_1 = 2\tau_o = 0.08$, $H_o = 10^5$.

The dimensionless variables used as follow

$$
\begin{gathered}
r' = \left(\frac{\lambda + 2\mu + \mu_e H_o^2}{\rho}\right)^{1/2} \frac{r}{k} \\
u' = \left(\frac{\lambda + 2\mu + \mu_e H_o^2}{\rho}\right)^{1/2} \frac{u}{k} \\
t' = \left(\frac{\lambda + 2\mu + \mu_e H_o^2}{\rho}\right) \frac{t}{k} \\
(\tau_o', \tau_1') = \left(\frac{\lambda + 2\mu + \mu_e H_o^2}{\rho}\right) \frac{(\tau_o, \tau_1)}{k} \\
q' = \frac{k}{k_o T_o} \left(\frac{\rho}{\lambda + 2\mu + \mu_e H_o^2}\right)^{1/2} q \\
R' = \left(\frac{\lambda + 2\mu + \mu_e H_o^2}{\rho}\right)^{1/2} \frac{R}{k} \\
\vartheta' = \frac{\vartheta}{T_o}, \quad \sigma' = \frac{\sigma}{\mu} \quad \sigma^{*\prime} = \frac{\sigma^*}{\mu}, \quad K_1' = K_1 T_o
\end{gathered}
\tag{31}
$$

Using Eqs. (31), (25) and (26) take the following forms

$$
\nabla^2 \vartheta = \left(1 + \tau_o \frac{\partial}{\partial t}\right) \dot{\vartheta} + g\left(1 + \tau_o \delta \frac{\partial}{\partial t}\right) \dot{e} \tag{32}
$$

$$
\ddot{e} = \nabla^2 e - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \times \left[\frac{a}{\sqrt{1 + 2K_1 \vartheta}} \nabla^2 \vartheta + \frac{a K_1}{(1 + 2K_1 \vartheta)^{3/2}} \left(\frac{\partial \vartheta}{\partial r}\right)^2\right] \tag{33}
$$

$$
\sigma_{rr} = \beta^2 \frac{\partial u}{\partial r} + (\beta^2 - 2)\frac{u}{r} - b\left(\frac{-1 + \sqrt{1 + 2K_1 \vartheta}}{K_1}\right) \tag{34}
$$

$$
\sigma_{\psi\psi} = (\beta^2 - 2)\frac{\partial u}{\partial r} + \beta^2 \frac{u}{r} - b\left(\frac{-1 + \sqrt{1 + 2K_1 \vartheta}}{K_1}\right) \tag{35}
$$

$$
\sigma_{zz} = (\beta^2 - 2)e - b\left(\frac{-1 + \sqrt{1 + 2K_1 \vartheta}}{K_1}\right) \tag{36}
$$

where

$$
b = \frac{\gamma T_o}{\mu}, \quad g = \frac{\gamma T_o k}{K_o}, \quad \beta = \left(\frac{\lambda + 2\mu}{\mu}\right)^{1/2}
$$

$$
a = \frac{\mu b}{\lambda + 2\mu + \mu_e H_o^2}
$$

We will use the boundary conditions on the internal surface, $r = R_1$ and the outer surface $r = R_2$ which are given by

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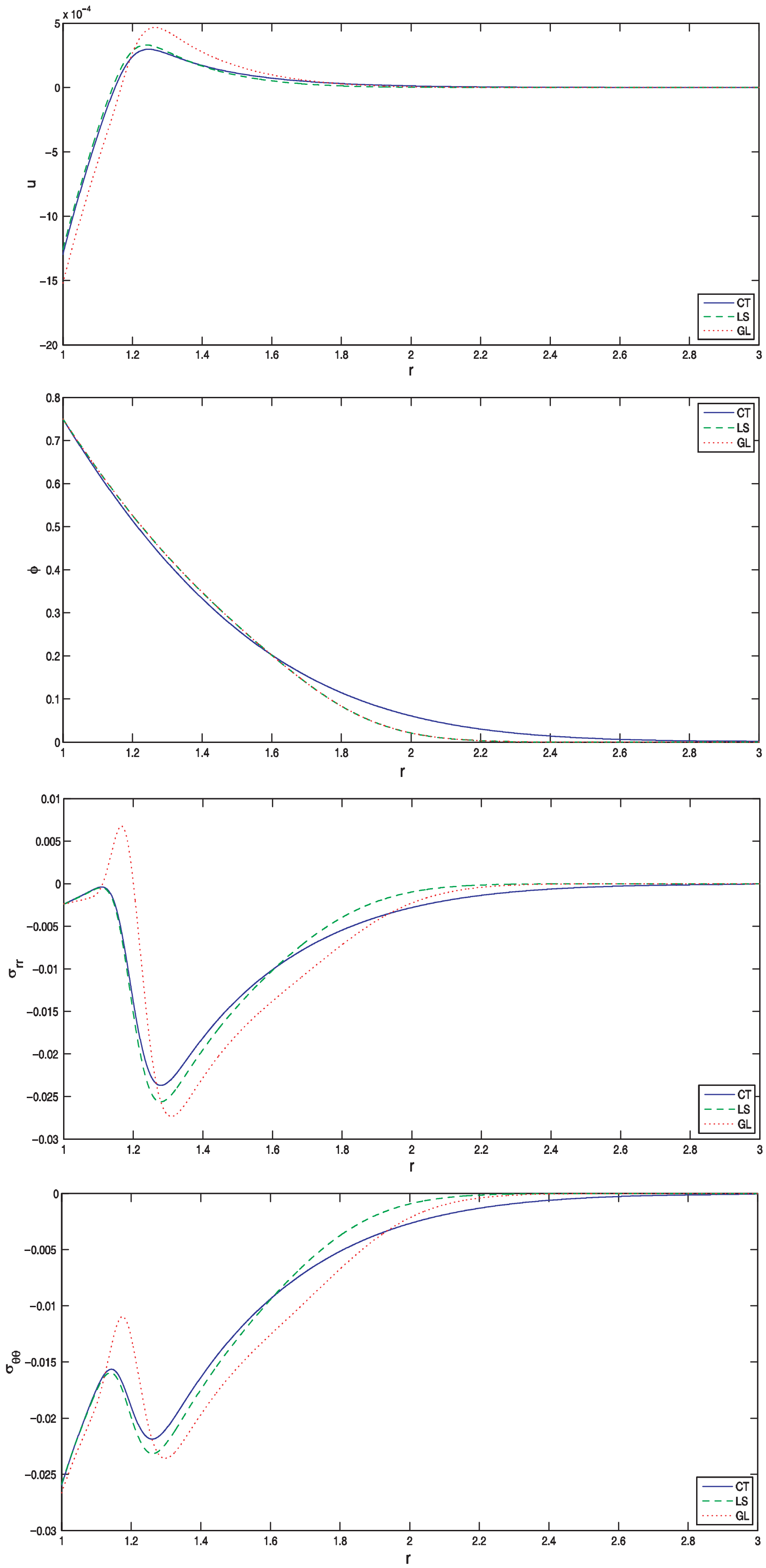

Fig. 2. Continued.

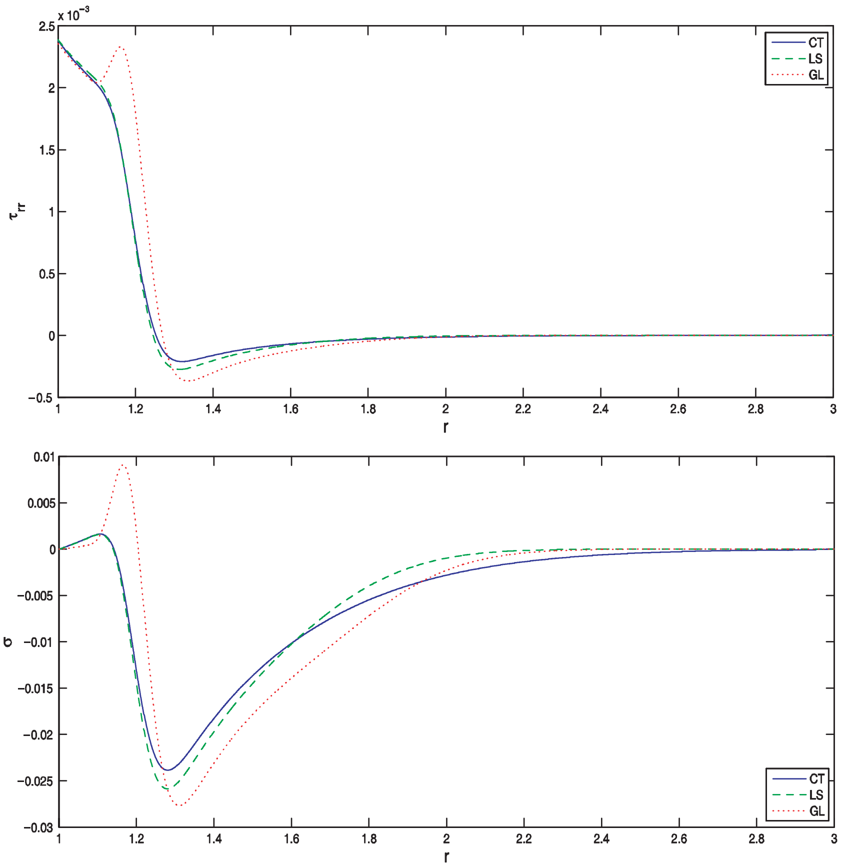

Fig. 2. Variations of CT, LS and GL theories on u, $\phi = \vartheta$, σ_{rr}, $\sigma_{\theta\theta}$, τ_{rr} and $\sigma = \sigma_{rr} + \tau_{rr}$ when $\tau_1 = 2\tau_o = 0.08$, $H_o = 10^5$.

(i) The internal surface $r = R_1$ is subjected to a thermal shock in the form

$$\theta(R, t) = \theta_o M(t)$$
$$\vartheta(R, t) = \delta M(t) \tag{37}$$

where

$$\delta = \left(1 + \frac{K_1}{2}\theta_o\right)\theta_o \tag{38}$$

(ii) The outer surface $r = R_2$ we have not any heat flux. We will use the generalized Fourier law of heat conduction, namely

$$q_r + \tau_o \frac{\partial q_r}{\partial t} = -K(\theta)\frac{\partial \theta}{\partial r} \tag{39}$$

By using Eq. (16), we have

$$q_r + \tau_o \frac{\partial q_r}{\partial t} = -K_o \frac{\partial \vartheta}{\partial r} \tag{40}$$

After using the non-dimensional variables, the last equation will take the form

$$q_r + \tau_o \frac{\partial q_r}{\partial t} = -\frac{\partial \vartheta}{\partial r} \tag{41}$$

Now, by using the boundary condition at $r = R_2$ which we have $q_r = 0$, then we get

$$\frac{\partial \bar{\vartheta}(R_2, s)}{\partial r} = 0 \tag{42}$$

The mechanical boundary conditions:
The internal and the outer surfaces $r = R_1$ and $r = R_2$ is traction free i.e.,

$$(\sigma_{rr} + \tau_{rr})(R_1, t) = 0$$
$$(\sigma_{rr} + \tau_{rr})(R_2, t) = 0 \tag{43}$$

where,

$$\tau_{rr} = \mu_e H_o^2 e \tag{44}$$

$$e = \frac{\partial u}{\partial r} + \frac{u}{r} \tag{45}$$

4. FINITE ELEMENT METHOD

A finite element scheme is used here to get the temperature and radial displacement. The Finite element method is a powerful technique originally developed for numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. On the other hand, the finite element method in different generalized thermoelastic problems has been applied by many authors (see for instant Abbas et al.[49–55]). The finite element method

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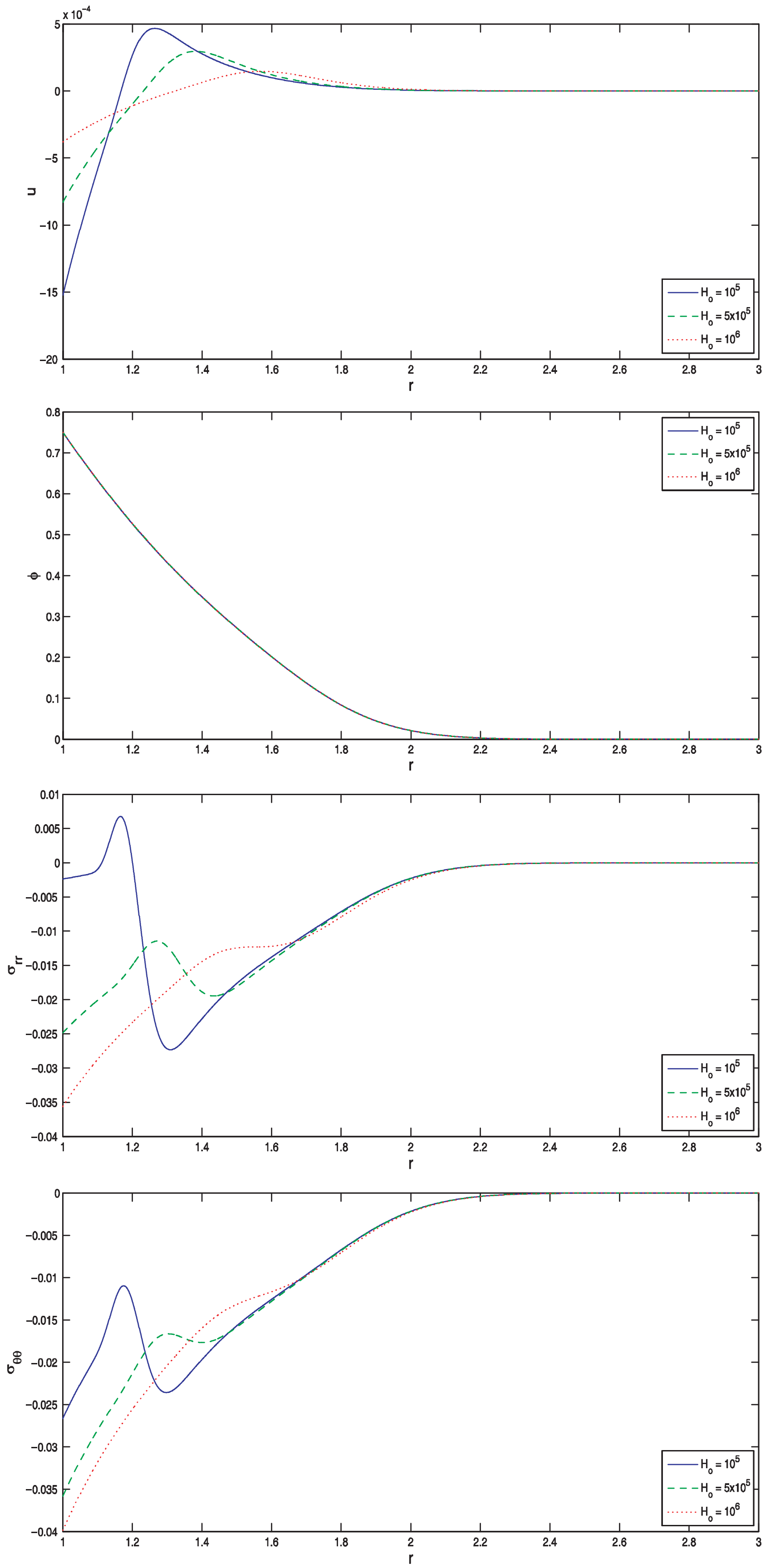

Fig. 3. Continued.

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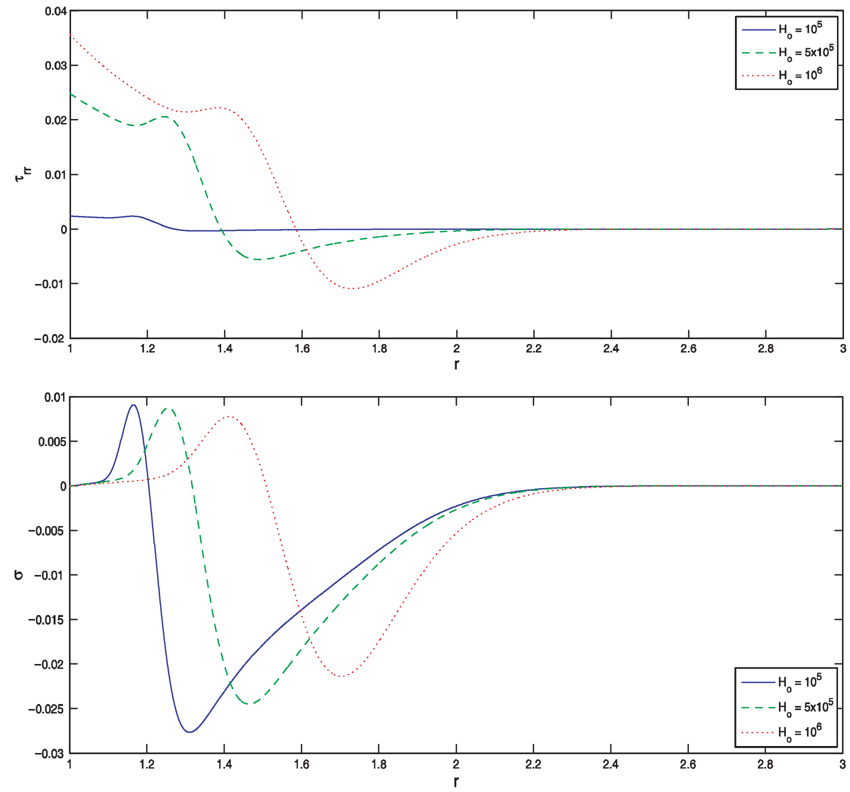

Fig. 3. Effect of the magnetic field on u, $\phi = \vartheta$, σ_{rr}, $\sigma_{\theta\theta}$, τ_{rr} and $\sigma = \sigma_{rr} + \tau_{rr}$ when $\tau_1 = 2\tau_o = 0.08$, $k_1 = -0.5$.

(FEM) in Refs. [31–33] is adopted due to its flexibility in modeling layered structures and its capability in obtaining full field numerical solution to investigate the thermo-mechanical shock problem of generalized thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity and magnetic field. The governing Eqs. (32) and (33) are coupled with initial and boundary conditions. The numerical values of the dependent variables like displacement u and the mapping of temperature ϑ are obtained at the interesting points which are called degrees of freedom. The weak formulations of the nondimensional governing equations are derived. The set of independent test functions to consist of the displacement δu and the mapping of temperature $\delta\vartheta$ is prescribed. The governing equations are multiplied by independent weighting functions and then are integrated over the spatial domain with the boundary. Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allows for the application of the boundary conditions. The same shape functions are defined piecewise on the elements. Using the Galerkin procedure, the unknown fields u and ϑ and the corresponding weighting functions are approximated by the same shape functions. The last step towards the finite element discretization is to choose the element type and the associated shape functions. Three nodes of quadrilateral elements are used. The shape function is usually denoted by the letter N and is usually the coefficient that appears in the interpolation polynomial. A shape function is written for each individual node of a finite element and has the property that its magnitude is 1 at that node and 0 for all other nodes in that element. We assume that the master element has its local coordinates in the range $[-1, 1]$. In our case, the one-dimensional quadratic elements are used, which given by linear shape functions

$$N_1 = \frac{1}{2}(1-\xi), \quad N_2 = \frac{1}{2}(1+\xi)$$

quadratic shape functions

$$N_1 = \frac{1}{2}(\xi^2 - \xi), \quad N_2 = 1 - \xi^2, \quad N_3 = \frac{1}{2}(\xi^2 + \xi)$$

On the other hand, the time derivatives of the unknown variables have to be determined by Newmark time integration method with 0.01 as time step.[31] In our investigation, we prepared the programs for finite element method by using Matlab software. After obtaining ϑ, the temperature increment θ can be obtained by solving Eq. (21).

5. NUMERICAL RESULTS AND DISCUSSION

For purposes of numerical evaluations, the copper material was chosen. The constants of the material were taken as[29]

$$K_o = 386\ \mathrm{Km \cdot m \cdot K^{-1}\ s^{-3}}, \quad \alpha_t = 1.78 \times 10^{-5}\ \mathrm{K^{-1}}$$
$$\rho = 8954\ \mathrm{kg \cdot m^{-3}}, \quad T_o = 293\ \mathrm{K}$$

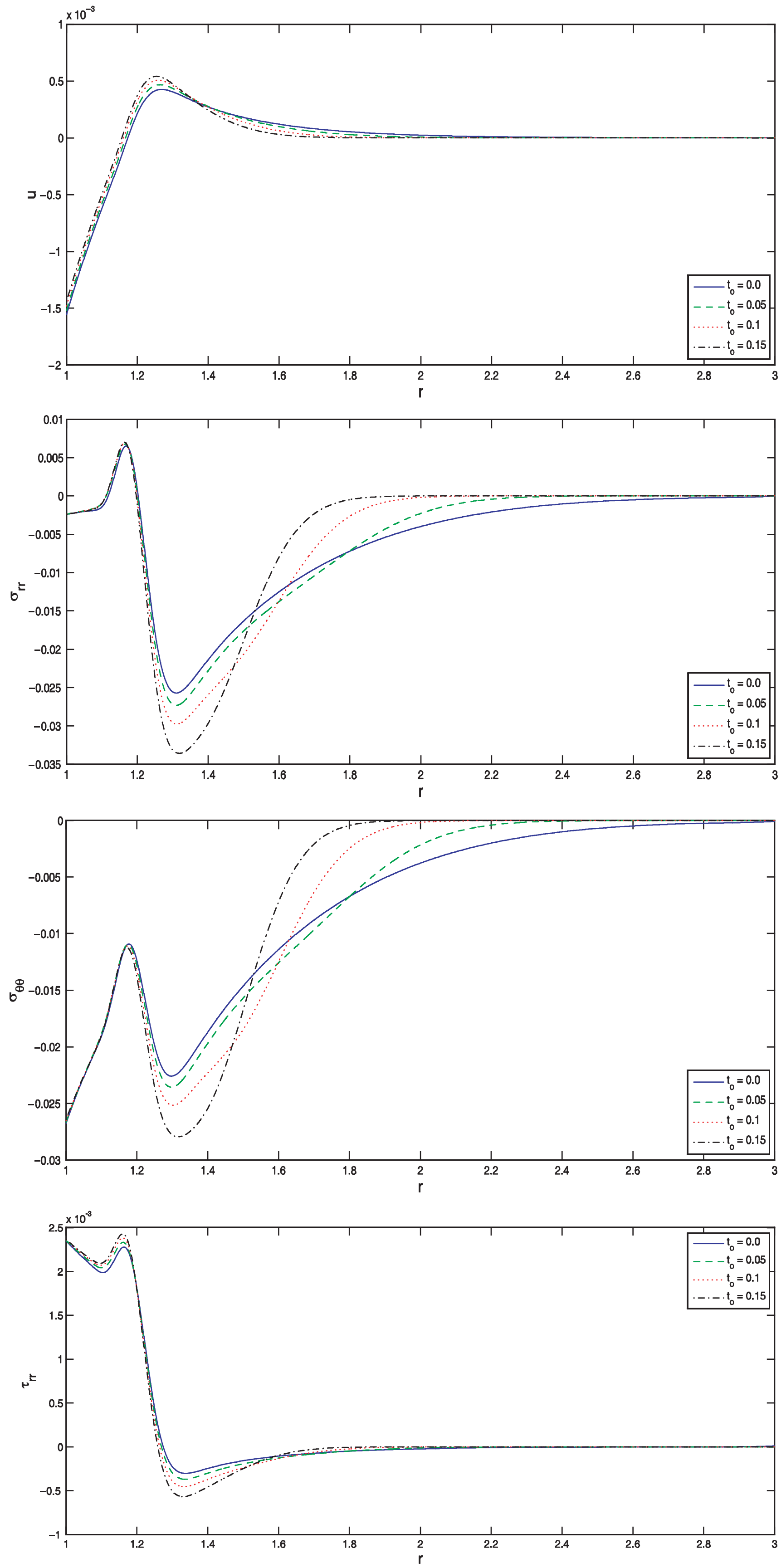

Fig. 4. Continued.

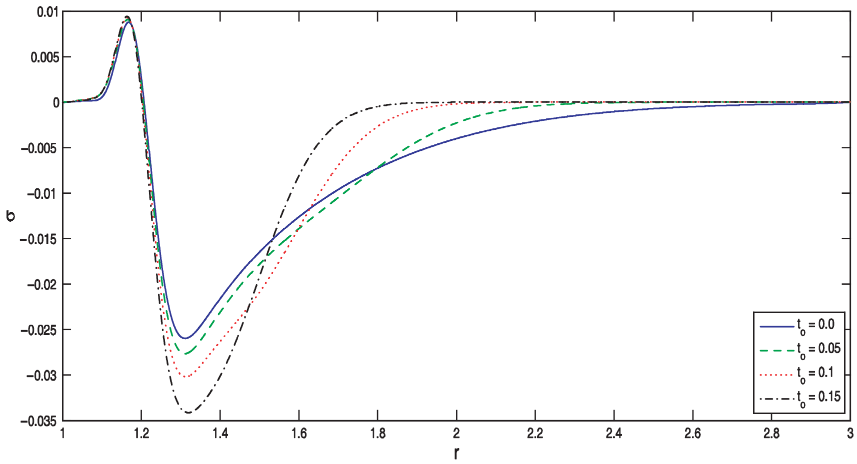

Fig. 4. Effect of the relaxation times on u, σ_{rr}, $\sigma_{\psi\psi}$, τ_{rr} and $\sigma = \sigma_{rr} + \tau_{rr}$ when $\tau_1 = 2\tau_o$, $H_o = 10^5$, $k_1 = -0.5$.

$$C_e = 383.1 \text{ m}^2 \text{ K}^{-1} \cdot \text{s}^{-2}, \quad \mu = 3.86 \times 10^{10} \text{ kg} \cdot \text{m}^{-1} \text{ s}^{-2}$$
$$\lambda = 7.76 \times 10^{10} \text{ kg} \cdot \text{m}^{-1} \text{ s}^{-2}, \quad \beta = 2, \quad \text{b} = 0.042$$
$$g = 1.61, \quad a = 0.0105$$

Before going to the analysis the grid independence test has been conducted and the results are presented in Table I. The grid size has been refined and consequently the values of different parameters as observed from Table I get stabilized. Further refinement of mesh size over 9000 elements does not change the values considerably, which is therefore accepted as the grid size for computing purposed.

Figures 1–4, show the variation of variable conduction, CT, LS and Gl theories, magnetic field and thermal relaxation times on u, $\phi = \vartheta$, σ_{rr}, $\sigma_{\theta\theta}$, τ_{rr} and $\sigma = \sigma_{rr} + \tau_{rr}$, if $\tau_1 = 2\tau_o = 0.08$, $k_1 = -0.5$, $H_o = 10^5$.

From Figure 1, it is clear that the displacement u increases with the increased values of the radius r and tends to zero as r tends to infinity; also, it is shown that u decreases with an increasing of the small values of k_1 and increases with the high values of k_1. It is obvious that the temperature ϕ decreases with an increasing of r but increases with an increasing of k_1. The radial and hoop stresses σ_{rr} and $\sigma_{\theta\theta}$ increase, decrease and tend to zero as r tends to infinity also it is clear that the stresses decreases with the increasing of k_1 but the Maxwell radial stress τ_{rr} takes inverse behavior respect to the mechanical stresses due to the normal direction for the magnetic field on the motion, it is display also that the total radial stress for the mechanical and Maxwell stress increases and decreases with an increasing of r and k_1.

From Figure 2, it is seen that GL model is very origin comparing with CD and LS models for determining the displacement, temperature and stresses.

From Figure 3, it is concluded that the displacement u increases for the small values of r with an increasing of magnetic field H_o, decreases and increasing with the large values of the radius r and tends to zero as r tends to infinity, it is obvious that the temperature doesn't affect with the variation of the magnetic field. The radial and hoop stresses σ_{rr} and σ_θ increase, decrease and tends to zero as r tends to infinity also it is clear that the stresses decreases with the small values of H_o and then increase and inclined with the large values of the radial r but the Maxwell radial stress τ_{rr} takes inverse behavior respect to the mechanical stresses also due to the normal direction for the magnetic field on the motion, it is display also that the total radial stress for the mechanical and Maxwell stress increases, decreases and tends to zero with an increasing of r but decreases with an increasing of H_o.

From Figure 4, it is clear that u increases with an increasing of r and increases for the small values of r with an increasing of relaxation times τ_o and τ_1, decreases with the large values of the radius r. The radial and hoop stresses σ_{rr} and $\sigma_{\theta\theta}$ increase, decrease and tends to zero as r tends to infinity also it is shown that the stresses increases with the small values of the relaxation times and then decrease and increase with the large values of the radial r but the Maxwell radial stress τ_{rr} and the total radial stress σ for the mechanical and Maxwell stress decreases, increases and tends to zero as r tends to infinity, it is obvious that τ_{rr} increases, decreases and then increases with the large values of r with the variation of the relaxation times.

NOMENCLATURE

α_t is the interal thermal expansion coefficient,
γ $= \alpha_t\,(3\lambda + 2\mu)$,
θ is the temperature increment, $\theta = T - T_o$, $\left|\dfrac{T - T_o}{T_o}\right| \ll 1$,
λ and μ are Lame's constants,
μ_e is the magnetic permeability,
σ_{ij} are the components of stress tensor,
ρ is the density,
τ_o and τ_1 are the relaxation times,

ϑ is the mapping of θ,
C_e is specific heat per unit mass,
e_{ij} are the components of strain tensor,
H_o is the constant magnetic field,
$\vec{H}$ is the magnetic field vector,
$\vec{J}$ is the electric crrent density,
k is the diffusivity,
K is the thermal conductivity,
t is the time,
T_o is the reference temperature,
u_i are components of displacement vector.

References

1. J. Duhamel, *J. De L Ecole Polytechnique* 15, 1 **(1837)**.
2. F. Neumann, Vorlesungen Uber die Theorie der Elasticitat, Meyer, Brestau **(1885)**.
3. V. Danilovskaya, *Prikl Mat. Mekh.* 14, 316 **(1950)**.
4. M. A. Biot, Mechanics of Incremental Deformation, Wiley, New York **(1965)**.
5. C. C. Ackerman and W. C. Overton, Jr, *Phys. Rev. Lett.* 22, 764 **(1969)**.
6. R. J. Von-Gutfeld and A. H. Nethercot, Jr., *Phys. Rev. Lett.* 17, 868 **(1966)**.
7. R. A. Guyer and J. A. Krumhansi, *Phys. Rev. Lett.* 148, 778 **(1966)**.
8. B. Taylor, H. J. Marris, and C. Flbaum, *Phys. Rev. Lett.* 23, 416 **(1969)**.
9. S. J. Rogers, *Phys. Rev. Lett.* 3, 1440 **(1971)**.
10. H. E. Jackson and C. T. Walker, *Phys. Rev.* B-3, 1428 **(1971)**.
11. V. Narayanamurti and R. C. Dynes, *Phys. Rev. Lett.* 28, 1461 **(1972)**.
12. H. Lord and Y. Shulman, *J. Mech. Phys. Solids* 15, 299 **(1967)**.
13. A. E. Green and K. A. Lindsay, *J. Elasticity* 2, 1 **(1972)**.
14. W. Nowinski, Theory of Thermoelasticity with Applications, Sijthoff and Noordhoof Int., Netherlands **(1978)**.
15. W. Nowacki, Dynamic Problems of Thermoelasticity, edited by, P. H. Francis and R. B. Hetnarski, Noordhoff Leyden **(1975)**, Vol. 43, pp. 269–282.
16. H. H. Sherief, On generalized thermoelasticity, Ph. D. Thesis, University of Calgary, Canada **(1980)**.
17. A. E. Green and P. M. Naghdi, *J. Thermal Stresses* 15, 253 **(1992)**.
18. R. S. Dhaliwal and H. H. Sherief, *Quart. Appl. Math.* 33, 1 **(1980)**.
19. R. B. Hetnarski and J. Ignaczak, *J. Thermal Stresses* 22, 451 **(1999)**.
20. S. Eraby and E. Suhubi, *J. Thermal Stresses* 9, 279 **(1986)**.
21. J. Ignaczak, *J. Thermal Stresses* 8, 25 **(1985)**.
22. J. Ignaczak, *J. Thermal Stresses* 1, 41 **(1978)**.
23. M. Ezzat, *Int. J. Eng. Sci.* 33, 2011 **(1995)**.
24. N. M. El-Maghraby and H. M. Youssef, *J. Applied Mathematics and Computation* 156, 577 **(2004)**.
25. N. M. El-Maghraby and H. M. Youssef, *J. Applied Mathematics and Computation* (*AMC*) 120, 172 **(2005)**.
26. H. M. Youssef, N. M. El-Maghraby, and A. A. El-Bary, *Computational Mathematics and Modeling* 17, 243 **(2006)**.
27. H. Youssef, *Journal of Applied Mathematics and Mechanics* 26 **(2005)**.
28. H. Youssef, *J. Thermal Stresses* 28, 521 **(2005)**.
29. H. Youssef, *J. Archive App. Mech.* 75, 553 **(2006)**.
30. N. Noda, Thermal stresses in materials with temperature-dependent properties, Thermal Stresses I, edited by R. B. Hetnarski, North-Holland, Amsterdam **(1986)**.
31. O. C. Zienkiewicz and R. L. Taylor, The finite element method, Fluid Dynamics, 5th edn., Butterworth-Heinemann **(2000)**.
32. J. N. Reddy, An Introduction to the Finite Element Method, 2nd edn., McGraw-Hill, New York **(1993)**.
33. R. D. Cook, D. S. Malkus, and M. E. Plesha, Concepts and Applications of Finite Element Analysis, 3rd edn., John Wiley, New York **(1989)**.
34. L. Knopoff, *J. Geophys. Res.* 60, 441 **(1955)**.
35. P. Chadwick, Elastic waves propagation in a magnetic field, *Proceeding of the International Congress of Applied Mechanics*, Brusseles, Belgium **(1957)**, Vols. 143–153.
36. S. Kaliski and J. Petykiewicz, *Proc. Vibr. Probl.* 4, 1 **(1959)**.
37. M. A. Ezzat and H. M. Youssef, *Int. J. Solids Structures* 42, 6319 **(2005)**.
38. A. Baksi, R. K. Bera, and L. Debnath, *Int. J. Eng. Sci.* 43, 1419 **(2005)**.
39. H. M. Yousesef and I. A. Abbas, *CMST* 13, 95 **(2007)**.
40. H. Tianhu, S. Yapeng, and T. Xiaogeng, *Int. J. Eng. Sci.* 42, 809 **(2004)**.
41. A. N. Abd-Alla, A. A. Yahia, and S. M. Abo-Dahab, *Chaos, Solitons and Fractals* 16, 211 **(2003)**.
42. S. M. Abo-Dahab and R. A. Mohamed, *J. Vibration and Control* 16, 685 **(2010)**.
43. V. Sajfert, P. Mali, N. Bednar, N. Pop, D. Popov, and B. Tošic, *Quantum Matter* 1, 134 **(2012)**.
44. P. K. Bose, N. Paitya, S. Bhattacharya, D. De, S. Saha, K. M. Chatterjee, S. Pahari, and K. P. Ghatak, *Quantum Matter* 1, 89 **(2012)**.
45. T. Ono, Y. Fujimoto, and S. Tsukamoto, *Quantum Matter* 1, 4 **(2012)**.
46. A. M. Abd-Alla, S. M. Abo-Dahab, and T. A. Al-Thamali, *J. Comput. Theor. Nanosci.* 10, 10 **(2013)**.
47. S. M. Abo-Dahab and I. A. Abbas, *Applied Mathematical Modeling* 35, 3759 **(2011)**.
48. R. B. Hetnarski, Thermal Stresses, North-Holand, Amesterdam **(1986)**, Vol. 1, pp. 391–396.
49. I. A. Abbas, *Int. J. Thermophys.* 33, 567 **(2012)**.
50. I. A. Abbas and A. N. Abd-Alla, *Arch. Appl. Mech.* 78, 283 **(2008)**.
51. I. A. Abbas, and A. N. Abd-Alla, *Acta Phys. Pol., A* 119, 814 **(2011)**.
52. I. A. Abbas and M. I. Othman, *Int. J. Thermophys.* 33, 913 **(2012)**.
53. I. A. Abbas, *Journal of Physics* 1, 3 **(2012)**.
54. I. A. Abbas and M. Othman, *Chin. Phys. B* 21, 014601 **(2012)**.
55. I. A. Abbas, Rajneesh Kumar, and Vijay Chawla, *Chin. Phys. B* 21, 084601 **(2012)**.

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